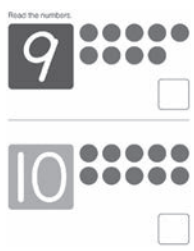
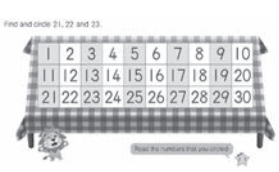
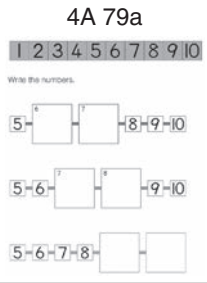
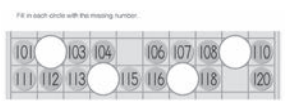
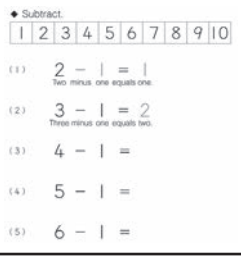
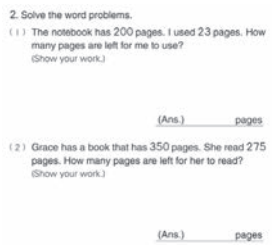
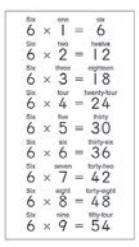
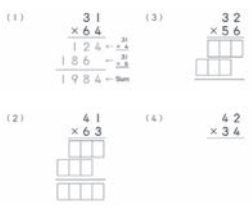
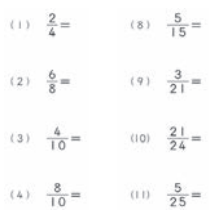
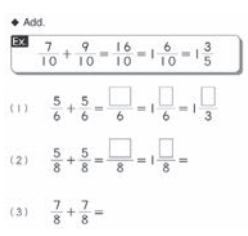
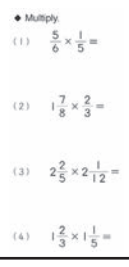
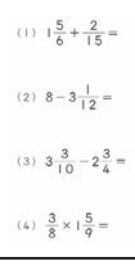
KUMON® DESCRIPTION OF STUDY LEVELS – MATH

Level	Samples	Highlights
6A	6A 1a  6A 55b  6A 149a 	Students become familiar with numbers up to 10 by counting pictures or dots and reading numbers. They should be able to read and easily recite numbers up to 10 and recognize the number of pictures or dots by counting.
5A	5A 11a  5A 113a  5A 183a 	Students read numbers and number tables up to 50 by counting pictures or dots, reading numbers or number tables, and finding the missing numbers in number tables. They should be able to easily recite numbers up to 50 and become familiar with numbers up to 100.
4A	4A 40a  4A 79a  4A 191b 	Students learn to write numbers up to 50. Students deepen their understanding of the number sequence by writing consecutive numbers and filling in the blanks in number tables, number boards, and complete-the-sequence problems. By the end of the level, students learn to read up to 100.
3A	3A 69b  3A 74a  3A 195a 	Students continue to enhance their understanding of the sequence of numbers as well as the number-writing skills they developed in Level 4A. Students are introduced to addition in Level 3A. At first, they master +1, +2, through +3 individually. The last 20 sheets of this level are dedicated to addition questions from +1 to +3.
2A	2A 31a  2A 78b  2A 200b 	In this level, students further develop basic mental calculation in addition by a sequential study of adding 4 through adding 10. It is very important that students master the contents of this level for smooth progress in subsequent levels. Level 2A aims to develop the concentration and work skills necessary for Level A.
A	A 68a  A 81a  A 191a 	Level A continues horizontal addition with larger numbers. Mastering addition ensures a smooth introduction to subtraction in this level. Level A aims to develop mental calculations while strengthening students' concentration ability and work skills.
B	B 80b  B 187a  B 199b 	In Level B, students study vertical addition and subtraction by using the mental calculation skills they have developed in previous levels. In the first half of Level B, students focus on addition of 2-digit numbers before studying addition of 3-digit numbers. Similarly, in the second half of the level, students study subtraction of 2-digit numbers before moving on to subtraction of 3-digit numbers.
C	C 31a  C 52a  C 200b 	Students study 2-digit by 1-digit multiplication based on their study of the multiplication tables in the first 50 worksheets of Level C. Next, students practice up to 4-digits by 1-digit multiplication. Finally, students are introduced to division by 1-digit numbers.
D	D 11a  D 91b  D 166a 	In Level D, students learn 2-digit by 2-digit multiplication before studying long division. At the end of the level, students study fractions: first learning to convert improper fractions into mixed numbers and vice versa, and then learning to reduce fractions.
E	E 21a  E 146a  E 199a 	In Level E, students learn to add, subtract, multiply and divide fractions. Borrowing from integers and efficient intermediate steps will help prepare students for more challenging computation of fractions in Level F.

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Level	Samples	Highlights
F	F 61a (1) $7-(8-3)$ $=7-\square$ $=$ (2) $2\times(5+4)$ $=$ (3) $2\times(6-2)$ $=$ F 134b (2) $3\frac{2}{5}+0.9=$ (3) $1.7-1\frac{1}{2}=$ (4) $4.2+\frac{3}{5}=4\frac{\square}{5}+\frac{3}{5}=$ F 176a (1) I read 40 pages of a book. This is equivalent to $\frac{2}{3}$ of the whole book. How many pages does this book have? (Show your work.) If the total number of pages is x pages, $\frac{2}{3}\times x=40$ $x=$ (Ans.) pages	Students continue calculations with fractions, now using the order of operations. Level F contains a challenging section of word problems, as well as work with decimals.
G	G 21a (1) $+3-1=$ (2) $-3+1=$ (3) $+4-3=$ G 98b (7) $-3^2-\frac{3}{4}\times\left(-2\frac{2}{3}\right)=$ (8) $0.25\div\left(\frac{2}{3}-1\frac{1}{4}\right)=$ (9) $0.12\times\left[1-\left(-\frac{2}{3}\right)^2\right]=$ G 182a (1) $\frac{1}{5}x-12=-\frac{1}{2}x-5$ [Sol] $\left(\frac{1}{5}x-12\right)\times\square=\left(-\frac{1}{2}x-5\right)\times\square$ (2) $\frac{1}{2}x-4=\frac{1}{3}x-1$	Students are introduced to positive and negative numbers and basic algebra. Students use their previously-learned four operations skills to master linear equations. Transposition and verification skills will lay the groundwork for advanced algebra.
H	H 24b (4) $\begin{cases}x-2y=-4\\-3x+2y=8\end{cases}$ (5) $\begin{cases}7x-2y=-25\\-5x+2y=19\end{cases}$ H 114a (1) $4xy\cdot7x^2=$ (2) $3x^2y\times1=$ (3) $4x^2y(-3xy)=$ (4) $-2x^3\times8x^2y=$ H 153b (12) $3x(a-b)+(a-b)=$ (13) $4a(x-y)-(x-y)=$ (14) $(x+y)+5a(x+y)=$ (15) $6y(x-2y)-5z(x-2y)=$	Students learn to solve simultaneous linear equations with two to four variables. Concepts of numerical and algebraic values are strengthened. Students are also introduced to transforming equations and factorization.
I	I 36b (11) $\sqrt{2}(\sqrt{2}+1)+\sqrt{8}=$ (12) $\sqrt{12}-\sqrt{3}(\sqrt{3}+2)=$ (13) $\sqrt{5}(\sqrt{10}-2)+\sqrt{45}=$ (14) $\sqrt{32}-\sqrt{3}(\sqrt{6}+\sqrt{12})=$ I 75b (5) $x^2+3x-1=0$ (6) $-x^2+7x+5=0$ I 161b (2) $y=x^2+6x+5$ ① Vertex (,) ② y-intercept (,) ③ x-intercepts (,) (,)	In Level I students learn about square roots, a critical skill needed for advancement to quadratic equations, which are also covered in the level. Students also learn about inequalities, linear functions and graphs, and quadratic functions and graphs. Level I concludes with advanced topics in geometry specifically related to the *Pythagorean Theorem*.
J	J 30a (3) $(a-2b)(3x-5y)+(2b-a)(x-y)$ $=$ (4) $4xy^2(3y-x)-2x^2(x-3y)^2$ $=$ J 115b (1) $(2x-y)+(y-x)i=1+3i$ (2) $(1+i)x-(1-2i)y=1+4i$ $(\quad)+(\quad)i=1+4i$ J 166b 2. When a polynomial $P(x)$ is divided by $x-3$, the remainder is 3. When it is divided by $x+4$, the remainder is -4. Find the remainder when $P(x)$ is divided by $(x-3)(x+4)$. J 196b 2. Prove the following inequalities. (1) Given $a+b=1$, prove the inequality $a^2+b^2>ab$. (2) Given $a+b=1$, prove the inequality $a+b>ab$.	Concepts learned through Levels H and I are expanded and reinforced in Level J. Students are introduced to advanced factoring methods, complex numbers, the discriminant, and the *Factor* and *Remainder* theorems. At the end of Level J, students conduct proofs of algebraic equalities and inequalities.
K	K 43b 2. Draw the graph of the given quadratic function, find the maximum and minimum values, and then, state the range. (the set of y values) that satisfies the domain. (1) $y=x^2+3$ $(-1\leq x\leq 2)$ (3) $y=-x^2+2x+1$ $(-1\leq x\leq 2)$ Maximum value: at $x=$ Minimum value: at $x=$ Range: $\leq y\leq$ K 138a 1. For each given fractional function, find the equations of the asymptotes and draw them on the given graphs. (1) $y=x+\frac{1}{x}$ (3) $y=x-1+\frac{1}{x-1}$ asymptotes: $x=$, $y=$ asymptotes: $x=$ K 183a Draw the graph of each of the following exponential functions. (1) $y=2^{-x+1}$ (2) $y=\left(\frac{1}{3}\right)^{x-1}$	Students acquire the basic properties of functions through a thorough study of quadratic functions. Level K introduces higher degree, fractional, irrational and exponential functions, and their corresponding graphs. The skills developed here will ease students into the calculus exercises of Level L.
L	L 14a Evaluate the following expressions. (1) $\log_3 36-4\log_{81}30+16\log_{81}\sqrt{15}$ $=$ (2) $\log_2 6\cdot\log_3 6-(\log_2 3+\log_2 2)$ $=$ (3) $\log_2 14\cdot\log_7 14-(\log_2 7+\log_2 2)$ $=$ L 42a Find the following limit values. (1) $\lim_{x\to-2}\frac{x^2-x-6}{x+2}=$ (2) $\lim_{x\to2}\frac{x^2+2x-8}{x-2}=$ (3) $\lim_{x\to-1}\frac{x^3+1}{x+1}=$ L 83b 1. Find the minimum value of the function $f(x)=x^3-3a^2x+a^3$ on the interval $0\leq x\leq 2$. Assume that $a>0$. [Sol] $f'(x)=3x^2-3a^2=3(x+a)(x-a)$ L 145a 1. Complete the following exercises. (1) Find the area, S_1, of the region enclosed by $y=-x^2+5x-2$, the x-axis, $x=1$ and $x=3$.	Students begin Level L by studying logarithmic functions and are introduced to calculus. Students study basic differentiation, and definite and indefinite integrals. The level concludes with an analysis of the applications of integration, including areas, volumes, velocity and distance.
M	M 4a Internal Division and External Dividing Points Given that point P lies on line segment AB, and AP : PB = m : n, line segment AB is said to be *internally divided by* P in the ratio m : n. Point P is called an *internal dividing point*. (m and n are positive numbers.) Ex Given points A(2, 0) and B(8, 6), find the coordinates of point P(x, y) which internally divides line segment AB in the ratio 2 : 1. 1. Given points A(-3, 0) and B(7, 8), find the coordinates of point P(x, 0) which internally divides line segment AB in the ratio 3 : 2. M 46a Ex Find the coordinates of the common points of two circles $x^2+y^2-10=0$ ... ① and $x^2+y^2-8x+4y+10=0$ ... ②. 2. Given that α is a natural number, prove the following equation by mathematical induction. $1\cdot2+2\cdot3+3\cdot4+\cdots+n(n+1)=\frac{1}{3}n(n+1)(n+2)$ M 103a Ex Place point P on the circumference of a circle with radius 1 such that general angle $\theta=45°$. Find the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$. M 175b 2. Convert the following expressions into a sum or difference using the formulas above.	In Level M, students begin by studying analytic geometry and loci. They will then study basics of trigonometric functions, graphs and inequalities, followed by more advanced trigonometric topics, including the Addition Formulas and Laws of Sines and Cosines.
N	N 2b General Term of an Arithmetic Sequence The general term of an arithmetic sequence $\{a_n\}$ with 1st term a and common difference d is $a_n=a+(n-1)d$ Ex Find the 1st term a and the general term of the arithmetic sequence $\{a_n\}$ whose common difference is -2 and 6th term is 8. 4. Find the 1st term a and the general term of the arithmetic sequence $\{a_n\}$ whose common difference is -3 and 11th term is -27. 5. Find the common difference d and the general term of the arithmetic sequence $\{a_n\}$ whose 1st term is 20 and 15th term is 90. N 52a 1. Given that n is a natural number, prove the following equation by mathematical induction. $1\cdot2+2\cdot3+3\cdot4+\cdots+n(n+1)=\frac{1}{3}n(n+1)(n+2)$ N 72a Find the following limits. Ex $\lim_{n\to\infty}\frac{1+3^n}{3^n+4^n}=\lim_{n\to\infty}\frac{\left(\frac{1}{4}\right)^n+\left(\frac{3}{4}\right)^n}{\left(\frac{3}{4}\right)^n+1}=0$ (1) $\lim_{n\to\infty}\frac{1+3^n}{2^n+3^n}=$ (2) $\lim_{n\to\infty}\frac{(-2)^n+2\cdot3^n}{3^n+1}=$ N 125a Find the following limits. (1) $\lim_{x\to0}\frac{2\tan x}{\sqrt{1+x}-1}=$ N 179a 1. Differentiate the following functions. (1) $y=\ln[e^x(1-x)]$	Students begin Level N by studying arithmetic, geometric, infinite and other various types of sequences and series. They will also study recurrence relations and mathematical induction. The level concludes with the topics of limits of functions and continuity, and the basics of differentiation.
O	O 1a Find the equation of the tangent to each given curve at point A. Ex $y=\sqrt{x}$, A(4, 2) [Sol] Let $f(x)=\sqrt{x}$. $f'(x)=\frac{1}{2\sqrt{x}}$ $f'(4)=\frac{1}{4}$ Therefore, the equation of the tangent is $y-2=\frac{1}{4}(x-4)$. $\therefore y=\frac{1}{4}x+1$ O 55a Find the following indefinite integrals. (1) $\int\frac{x^2+5x+12}{\sqrt{x}}dx=$ O 111a As with indefinite integrals, Integration by Parts is also true for definite integrals. Integration by Parts for Definite Integrals $\int_a^b f(x)g'(x)dx=\left[f(x)g(x)\right]_a^b-\int_a^b f'(x)g(x)dx$ This method is called *Integration by Parts for Definite Integrals*. Find the following definite integrals using Integration by Parts. Ex $\int_0^{\frac{\pi}{2}}x\cos x\,dx=\int_0^{\frac{\pi}{2}}x(\sin x)'dx$ ← Rearranging into the form $\int f'(x)g(x)dx$ $=\left[x\sin x\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}(x)'\sin x\,dx$ ← $\left[f(x)g(x)\right]_a^b-\int_a^b f'(x)g(x)dx$ $=\frac{\pi}{2}-\int_0^{\frac{\pi}{2}}\sin x\,dx$ $=\frac{\pi}{2}-\left[-\cos x\right]_0^{\frac{\pi}{2}}$ $=\frac{\pi}{2}-1$ O 163a Solve the following differential equations. Ex $\frac{dy}{dx}=x^2-x+2$ [Sol] Integrating both sides with respect to x, $y=\int(x^2-x+2)dx$ $=\frac{1}{3}x^3-\frac{1}{2}x^2+2x+C$ (C is an arbitrary constant) (1) $\frac{dy}{dx}=x^2-2x$ O 191a 1. Given function $f(x)=x^2e^{-\frac{x}{a}}$ $(a>0)$, solve the following questions. (1) When $f(x)$ has a relative maximum value at $x=c$, express c in terms of a.	In Level O, students first study advanced differentiation and applications of differential calculus, then continue with an in-depth study of advanced integration and its applications. The level concludes with differential equations and various application problems using calculus.
X	In Level X, students can choose to study elective topics: vectors, matrices, mapping and transformations, probability and statistics.	
	XV 67a 1. Given the fundamental vectors $\vec{e_1}=(1,0,0)$, $\vec{e_2}=(0,1,0)$, and $\vec{e_3}=(0,0,1)$, evaluate the following inner products. (1) $\vec{e_1}\cdot\vec{e_2}=$ (2) $\vec{e_1}\cdot\vec{e_3}=$ (3) $\vec{e_2}\cdot\vec{e_3}=$ 2. Given that the cube below has side length a, evaluate the following inner products. (1) $\overrightarrow{AB}\cdot\overrightarrow{AF}=$ (2) $\overrightarrow{BE}\cdot\overrightarrow{BG}=\overrightarrow{BE}\cdot\overrightarrow{AC}=$	XM 32a 1. Given that A, B, and C are all 2×3 matrices, where $B=\begin{bmatrix}1&8&-7\\-2&0&3\end{bmatrix}$, and given that O is a zero matrix, determine the matrices A and C which satisfy the following equations. $3A-4B+2C=O$ $2A-3B+C=O$ 2. Given that $A=\begin{bmatrix}5&0&-1\\-3&1&-3\end{bmatrix}$ and $B=\begin{bmatrix}0&5&7\\1&-2&1\end{bmatrix}$, determine the matrix C which satisfies the following equation. $2C-A=\frac{1}{3}[2B-(4A-C)]$	XP 30a 1. Given the letters a,a,a,b,b,b,c,c,c, and c, determine the number of different permutations of lining up the letters. 2. Given the numbers 1,2,3,4,5, and 6, determine how many different three-digit numbers can be formed, allowing repetitions.	XS 26a 1. Given the probability density function $y=f(x)$ for X shown below, complete the following exercises. (1) Determine the value of k. [Sol] $f(x)=\begin{cases}\frac{k}{2}x & (0\leq x\leq2)\\ \square & (2\leq x\leq4)\end{cases}$ $\therefore\int_0^4 f(x)dx=\int_0^2\frac{k}{2}x\,dx+\int_2^4\square\,dx$ $=$ (2) Evaluate $P(1\leq X\leq3)$. (3) Determine μ.

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